First we set $w = x^2/l$ (the familiar rough approximation for the fundamental bending mode). Then Eq. (4) reads

$$k_s \cdot 2 = k_s[0 + (4 - 2)] \quad (6)$$

This example does not support the Yu contention that the end terms may be left out. Of course, we used a poor approximation; let us try a better one. If the thermal field is weak, then the bending mode will be, almost, a natural bending mode w_n, and this we now assume

$$w(x) \approx w_n(x) \quad (7)$$

Using the boundary conditions

$$w_n(0) = w_n'(0) = w_n''(l) = w_n'''(l) = 0 \quad (8)$$

we find that Eq. (4) becomes

$$k_s\left[\frac{1}{2}\int_0^l (w_n'^2)'dx\right] = k_s\left\{\int_0^l w_n'''w_n dx + [w_n'^2(l) - 0]\right\}$$

whence

$$k_s[\tfrac{1}{2}w_n'^2(l)] = k_s\{-\tfrac{1}{2}w_n'^2(l) + [w_n'^2(l)]\} \quad (9)$$

Here the "simplification" Eq. (5) amounts to a straightforward reversal of the sign of the k_s term.

Now the term with k_s in Eq. (10) of Ref. 1 is the one term which matters for stability. This term is described by Eq. (9) to the degree that the approximation Eq. (7) is valid, and of this approximation Yu makes use later in his paper.** This fully explains why the Yu result is just the reverse of the correct answer.

Certain misunderstandings remain to be clarified which have arisen in connection with the original version of Ref. 1.[8] This version contains considerable matter which, as a patient reader would find out eventually, is not used in the actual analysis. It also contains some strange remarks.†† A hurried reader, let us call him B, does not readily find out what is actually being done. He prefers to make his independent analysis.

The point to be made here is twofold: 1) the two analytical approaches, the one chosen by Yu and the one chosen by B, are superficially similar, and, by coincidence, their results also look deceptively similar; and 2) there is in fact a very real difference.

B starts with the left-hand side of Eq. (1) (so does Ref. 8) and derives Eq. (2) for himself. Realizing that M_T, though written as a moment, is in fact a thermally induced curvature which does not involve structural forces, B does not try to transform M_T into a "force" by means of the double integration by parts in Eq. (1). He goes the direct way, writing

$$\left(\int_0^l M_T w''dx\right)_{\text{tracer}} = k_s\int_0^l w'w''dx = k_s\left(ww''|_0^l - \int_0^l ww'''dx\right) \quad (10)$$

This result, the right-hand side of Eq. (10), B compares with the Yu result, the right-hand side of Eq. (5).

B sees certain end terms in Eq. (10) which are not in Eq. (5). But, and this is the point, these end terms are unlike those in Eq. (4). The end terms in Eq. (10) disappear, all of them, when the natural approximation Eqs. (7) and (8) is introduced. B readily accepts that to leave out the end terms "simplifies greatly."

** Note that Eq. (9) implies that w_n''' is essentially negative. This fact Yu does notice (in the paragraph after Table 1) but he does not heed the warning which this fact implies.

†† Much of this is eliminated in Ref. 1. Still in Ref. 1 is the following remark: ". . . we do not consider booms which are nearly parallel to the sun ray." This remark eliminates, for no reason which this writer can see, just that angular range where instability is most pronounced (including the case, boom and ray parallel, which corresponds to the analysis of Augusti[4]). The author should explain.

This leaves B with the case of the missing minus sign. Not ready to suspect an error in an elementary analysis, B is ready to be persuaded that there is a wrong sign with the physical constant k_s. After all, action and reaction do get mixed up sometimes.

In the here relevant section II of his paper[5] for the ASME/AIAA SDM Conference (which section was a last minute addition) this writer, one of several B's, committed two sins for which he now wishes to apologize: 1) he reported his derivation (of a correct result) in a hasty manner which he does not wish to defend in detail; and 2) he stated that the sign error[8] was in the constant k_s.

At the Conference, J. D. Graham stated privately that he suspected the error source to be in a boundary condition. After the Conference, the error source was properly identified, and the subsequent letter exchange showed agreement with Graham.[9]

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Reply by Author to P. F. Jordan and G. Augusti and New Results of Two-Mode Approximation Based on a Rigorous Analysis of Thermal Bending Flutter of a Flexible Boom

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SOME questions have been raised since the author's original paper and its preprint form were published.[1] For the benefit of those interested in the problem of thermal bending flutter, a summary of their current status seems to be in or-

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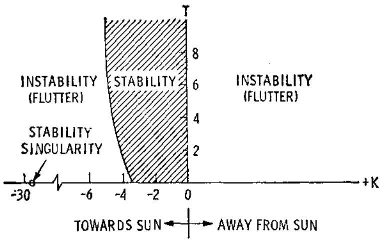

Fig. 1 Two-mode approximations.

der. Interestingly enough, all the questions are associated with the thermal curvature in one way or another.

The first of these was raised by Jordan,[2] who questioned the negative sign used by the author in the following expression relating the thermal moment M_T to the thermal curvature $\varkappa_T$†:

$$M_T = -EI\varkappa_T \tag{1}$$

The classical beam equation has been written in the form[1]

$$EIw'''' + M_T'' + \rho A\ddot{w} = 0 \tag{2}$$

or, on substitution of Eq. (1),

$$EIw'''' - EI\varkappa_T'' + \rho A\ddot{w} = 0 \tag{3}$$

Signs may be verified readily by applying Eq. (3) to statical thermal bending, for which the equation gives, with a nonessential linear expression neglected,

$$w'' = \varkappa_T \tag{4}$$

Equation (4) agrees with Fig. 1 of the original paper,[1] where the heat radiation is shown to generate both a positive $\varkappa_T$ and a positive second derivative w''. The negative sign in Eq. (1) is therefore correct, as is further concurred here by both Jordan and Augusti. It is also important to note that a proper discussion of the sign convention of M_T must be based on a simultaneous consideration of the foregoing Eqs. (1) and (2) and Fig. 1 of Ref. 1. As long as Eqs. (3) and (4) can be deduced with correct signs, there is no reason why the sign convention of M_T in Eqs. (1) and (2) cannot be reversed. (With the wrong sign in Jordan's governing equations[2] corrected, his result for stability[2] would be reversed and become the same as that given in Ref. 1.)

The second question deals with the thermal curvature itself. As now concurred by Augusti in his comments, the governing equation

$$\partial\varkappa_T/\partial t + \varkappa_T/\tau = K\cos(\alpha + \theta) \tag{5}$$

as originally proposed by Etkin and Hughes[3] and subsequently used by the author,[1] agrees essentially with that used in Augusti's own work.[4] The following exact series solution to Eq. (5) is further given by the author in Ref. 1:

$$\varkappa_T = k_c - k_s(w' - \tau\dot{w}' + \tau^2\ddot{w}' - \tau^3\dddot{w}' + \ldots) \tag{6}$$

the use of which in the thermal flutter problem has been explored in an Appendix to the paper.[1] In the main body of the paper, however, only the following truncated form is used:

$$\varkappa_T = k_c - k_s(w' - \tau\dot{w}') \tag{7}$$

which is also assumed by Jordan in his comments here. The use of this truncated form has not been and cannot be justified except for the fact that it leads to the same result concerning the effect of boom orientation on stability as the complete series form in Eq. (6) when a single cantilever mode is used to approximate the thermal deflection.

The third question is concerned with the further simplified expression for the thermal curvature,

$$\varkappa_T = k_c \tag{8}$$

which has been introduced[1] in the boundary integral in the variational equation of motion. This, together with the use of the (isothermal) modes of free vibration of a cantilever beam, makes all boundary conditions satisfied identically, thus preparing for the subsequent use of the usual Galerkin procedure.[1] However, the use of Eq. (8) in the boundary integral is inconsistent with the use of Eq. (7) in the field integral. That such usage is inadequate was first suspected by G. F. Banks of the NASA-Goddard Space Flight Center and later by J. D. Graham at the 1969 New Orleans meeting. Aside from advocating the single-degree-of-freedom approach,‡ Jordan deals mainly with this aspect of the thermal flutter problem in his comments here.

To see how stability is affected by the simplifying assumption (8), we start with the complete variational equation of motion, Eq. (3) of Ref. 1. By neglecting the tip mass and inserting M_T from Eqs. (1) and (7), the variational equation takes the form

$$\int_{t_0}^{t_1} dt \int_0^l \{w'''' + k_s(w''' - \tau\dot{w}''') + C\ddot{w}\}\delta w dx + \int_{t_0}^{t_1} [\{w'' - k_c + \underline{k_s(w' - \tau\dot{w}')}\}\delta w' - \{w''' + \underline{k_s(w'' - \tau\dot{w}'')}\}\delta w]_{x=0}^{x=l} \quad dt = 0 \tag{9}$$

The adoption of Eq. (8) in the boundary integral eliminates the underlined terms, as may be seen by comparing the complete and simplified variational equations, Eq. (9) here and Eq. (10) of Ref. 1. When a single-mode approximation is used, Eq. (9) yields a characteristic equation that is quadratic, and the effect of boom attitude on thermal flutter is determined by the sign of the coefficient of the linear term,§ which comes from the $k_s\tau$-terms in Eq. (9). If $W(x)$ represents the form of the single mode, the coefficient may be shown to be proportional to

$$K = \int_0^l W''' W dx + \underline{[(W')^2 - W''W]_{x=0}^{x=l}} \tag{10}$$

Again, the underlined terms become absent if a constant thermal curvature is assumed in the boundary integral.

To seek an approximate solution to Eq. (9), any reasonable mode function may be used, in accordance with what Houbolt and Brooks[5] have called the modified Galerkin procedure. Thus, when a single cantilever mode is used, as in the original paper,[1] we have $W(x) = X_n(x)$, and Eq. (10) becomes

$$K = \int_0^l X_n'''X_n dx + \underline{[X_n'(l)]^2}$$

Since, by integration by parts,

$$\int_0^l X_n'''X_n dx = -\tfrac{1}{2}[X_n'(l)]^2 \tag{11}$$

K is finally

$$K = -\tfrac{1}{2}[X_n'(l)]^2 + \underline{[X_n'(l)]^2}$$

With Eq. (8) adopted in the boundary integral, the underlined term vanishes, and

$$K_{\text{simplified}} = -\tfrac{1}{2}[X_n'(l)]^2$$

which is the same as before. [See the original characteristic equation, Eq. (15) of Ref. 1, where the coefficient of λ involves I_{nn}, which is by definition proportional to the integral in Eq. (11) here.] On the other hand, without such simplification,

† Unless otherwise specified, notations in this Reply are the same as those in the original paper.[1]

‡ The work of R. M. Beam quoted by Jordan also deals with a one-degree-of-freedom system in which the boom acts as a massless torsional spring.

§ The sign of the constant term in the characteristic equation governs the possibility of having divergence or static instability, in contrast to flutter or dynamic instability.

the underlined term must be retained, and we have

$$K_{\text{complete}} = +\tfrac{1}{2}[X_n'(l)]^2 \qquad (12)$$

The difference in sign in these two results leads to a similar sign reversal of the linear term in the characteristic equation, which in turn reverses the original result concerning the effect of boom attitude on flutter. The correct result based on a *single-mode approximation* should be that *thermal flutter can take place only when the boom is pointed away from the sun, but not when it is towards the sun.*

It will be of interest to examine two more cases of single-mode approximation as suggested by Houbolt.[6] In the first of these the mode function is taken as $W(x) = x^2$, for which Eq. (10) yields

$$K = 0 + \underline{2l^2}$$

i.e., K is positive just as in Eq. (12), and the linear term in the characteristic equation vanishes altogether if Eq. (8) is adopted in the boundary integral. In the second case, the mode function is assumed to be $W(x) = x^3$. We then have

$$K = 1.5l^4 + \underline{3l^4} \qquad (13)$$

which is still positive, and for this case the adoption of Eq. (8) in the boundary integral does not affect the result on flutter, since K is always positive, with or without the underlined term.

New Results of Two-Mode Approximation Based on a Rigorous Analysis¶

Thus far, Eq. (7) has been used extensively, and much has been based on single-mode approximations. However, as already noted, the use of the truncated form given by Eq. (7) cannot be justified, except for the fact that it leads to the same result concerning the effect of boom orientation on stability as the complete series form in Eq. (6) when a single cantilever mode is used. The adoption of a single mode in the stability analysis of a continuous body constitutes another major assumption whose validity can be challenged. To remove possible restrictions imposed by these assumptions, additional work has been carried out with an attempt to solve rigorously the problem of thermal bending flutter, by making exclusive use of Eq. (6) and the complete infinite series of cantilever modes.

To make possible the effective and rigorous use of isothermal modes of free vibration of a beam, the analysis is based on a modified form of the variational equation of motion, which is derived first. The use of the complete infinite series of free modes of a beam then leads to an exact characteristic equation of the infinite degree. It will be seen that when *two cantilever modes* are used, *thermal flutter can take place when the boom is pointed either away from or towards the sun.* No effort will be made to discuss here the effects of a tip mass, viscoelastic damping, and a viscous damper. However, studies of such effects can be carried out readily by means of the techniques developed in Ref. 1, which may still serve some useful purpose.

Since equations for the general case of coupled flexure and torsion were derived elsewhere,[7] the details for deriving from Hamilton's principle the variational equation of uncoupled flexural motion were omitted from Ref. 1, but they are needed in the formulation of a modified form of the variational equation. Neglecting the tip mass, we have

$$\delta \int_{t_0}^{t_1} T dt = \delta \int_{t_0}^{t_1} dt \int_0^l \tfrac{1}{2}\rho A \dot{w}^2 dx = -\int_{t_0}^{t_1} dt \int_0^l \rho A \ddot{w}\delta w dx \qquad (14)$$

¶ These results have been briefly reported by the author in the invited general lecture, "Some Problems in Astroelasticity," at the Fifth Southeastern Conference on Theoretical and Applied Mechanics, April 1970.

$$\delta \int_{t_0}^{t_1} U dt = \delta \int_{t_0}^{t_1} dt \int_0^l (\tfrac{1}{2}EIw''^2 + M_T w'')dx = \int_{t_0}^{t_1} dt \int_0^l (EIw''\delta w'' + M_T\delta w'')dx \qquad (15)$$

When Eq. (15) is integrated by parts, and Eqs. (14) and (15) are substituted into Hamilton's principle, the original variational equation of motion[1] is obtained:

$$\int_0^{t_1} dt \int_0^l (EIw'''' + M_T'' + \rho A\ddot{w})\delta w dx + \int_{t_0}^{t_1} [(EIw'' + M_T)\delta w' - (EIw''' + M_T')\delta w]_{x=0}^{x=l} \quad dt = 0 \qquad (16)$$

Let us now integrate by parts only the first term in Eq. (15), so that

$$\delta \int_{t_0}^{t_1} U dt = \int_{t_0}^{t_1} dt \int_0^l EIw'''' \,\delta w dx + \int_{t_0}^{t_1} [EIw''\delta w' - EIw'''\delta w]_{x=0}^{x=l} \quad dt + \int_{t_0}^{t_1} dt \int_0^l M_T \delta w'' dx \qquad (17)$$

Hamilton's principle then yields

$$\int_{t_0}^{t_1} dt \int_0^l (EIw'''' + \rho A\ddot{w})\delta w dx + \int_{t_0}^{t_1} dt \int_0^l M_T\delta w'' dx + \int_{t_0}^{t_1} [EIw''\delta w' - EIw'''\delta w]_{x=0}^{x=l} \quad dt = 0 \qquad (18)$$

which will be called the modified variational equation of motion. In the usual manner the field equation and appropriate boundary conditions for the thermal problem can be written from the field and boundary integrals, respectively, of Eq. (16).[1] In contrast, these cannot be deduced from Eq. (18), because the second field integral associated with the thermal effect is still involved with $\delta w''$ but not δw. Intentionally, integration by parts is not carried out for this integral, so that the thermal effect is isolated and does not appear in the other integrals. In particular, the boundary integral in Eq. (18) is different from that in Eq. (16), in that the former now has the usual form for the isothermal case. This enables us to make effective use of the usual solutions for free vibrations of a beam which will make the boundary integral in Eq. (18) vanish exactly, just as in the isothermal case. Such solutions will not make the two field integrals in the same equation also vanish exactly, but direct integration with respect to x can be carried out. Since the infinite number of solutions of free vibrations of a given beam form a complete series, a rigorous solution to the thermal problem in the form of an infinite series is obtainable. This approach based on Eq. (18) apparently is new and can be extended to the general problem of coupled thermal bending and twisting of a thin-walled open section.

Equations (1) and (6) are next substituted in Eq. (18) to yield

$$\int_{t_0}^{t_1} dt \int_0^l (w'''' + C\ddot{w})\delta w dx - \int_{t_0}^{t_1} dt \int_0^l [k_c - k_s(w' - \tau\dot{w}' + \tau^2\ddot{w}' - \tau^3\dddot{w}' + \ldots)]\delta w'' dx + \int_{t_0}^{t_1} [w''\delta w' - w'''\delta w]_{x=0}^{x=l} \quad dt = 0 \qquad (19)$$

As has been pointed out, the boundary integral now has the same form as in the isothermal case. It can be made to vanish by taking

$$w = \sum_{n=1}^{\infty} X_n(x)T_n(t) \qquad (20)$$

because the free modes $X_n(x)$ satisfy all boundary conditions of the beam in the isothermal case. The field integrals in

Eq. (19), on the other hand, can now be integrated with respect to x. This leads to the following governing equation for T_n:

$$C\omega_n^2 T_n + C\ddot{T}_n - \sum_{m=1}^{\infty} I_{nm}k_s(T_m - \tau\dot{T}_m + \tau^2\ddot{T}_m - \tau^3\dddot{T}_m + \ldots) = (1/l)k_c X_n'(l) \quad (n = 1,2,\ldots) \quad (21)$$

For stability analysis, the nonhomogeneous part on the right-hand side of Eq. (21), which is essentially a forcing function, is ignored. By further taking

$$T_n(t) = A_n e^{\lambda t} \quad (22)$$

Eq. (21) yields

$$C(\omega_n^2 + \lambda^2)A_n - \sum_{m=1}^{\infty} I_{nm}k_s(1 - \tau\lambda + \tau^2\lambda^2 - \tau^3\lambda^3 + \ldots)A_m = 0 \quad (n = 1,2,\ldots) \quad (23)$$

Since the infinite series in the parentheses is equal to $(1 + \tau\lambda)^{-1}$, the result becomes

$$\zeta_n C A_n - k_s \sum_{m=1}^{\infty} I_{nm}A_m = 0 \quad (n = 1,2,\ldots)$$

with $\zeta_n = (\tau\lambda^3 + \lambda^2 + \omega_n^2\tau\lambda + \omega_n^2)$, from which the following characteristic equations can be written:

$$\begin{vmatrix} \zeta_1 C - k_s I_{11} & -k_s I_{12} & \ldots \\ -k_s I_{21} & \zeta_2 C - k_s I_{22} & \ldots \\ \ldots & \ldots & \ldots \end{vmatrix} = 0 \quad (24)$$

Equation (24) is exact if an infinite number of modes are used. When a *single mode* is used as an approximation, it reduces to

$$\zeta_n C - k_s I_{nn} = 0 \quad (n = 1,2,\ldots) \quad (25)$$

Thermal flutter is governed by the Routh discriminant of Eq. (25), which is easily shown to be $R = \tau k_s I_{nn}/C$. Since I_{nn} is negative for a cantilever mode, we have $R \lessgtr 0$ if $k_s \gtrless 0$. Thermal flutter will thus take place if $k_s > 0$, that is, if the boom is pointed away from the sun. This result agrees with the results given by the other authors[4,8] based on single-degree-of-freedom discrete models.

When *two cantilever modes* are used, say, the first and second modes, the characteristic equation can be written from Eq. (24) as

$$B_0\Lambda^6 + B_1\Lambda^5 + B_2\Lambda^4 + B_3\Lambda^3 + B_4\Lambda^2 + B_5\Lambda + B_6 = 0 \quad (26)$$

where the coefficients are

$$B_0 = c_1^2T^2, \quad B_1 = 2c_1^2T$$

$$B_2 = c_1^2 + (c_1 + c_2)c_1T^2$$

$$B_3 = [2(c_1 + c_2) - K(c_{11} + c_{22})]\, c_1T$$

$$B_4 = [c_1 + c_2 - K(c_{11} + c_{22})]c_1 + c_1c_2T^2$$

$$B_5 = [2c_1c_2 - K(c_1c_{22} + c_2c_{11})]T$$

$$B_6 = c_1c_2 - K(c_1c_{22} + c_2c_{11}) + K^2(c_{11}c_{22} - c_{12}c_{21})$$

and the dimensionless parameters are $\Lambda = \lambda/\omega_1$, $T = \omega_1\tau$, $K = k_s l$, $c_1 = C\omega_1^2 l^4$, $c_{11} = I_{11}l^3$, $c_{12} = I_{12}l^3$, $c_2 = C\omega_2^2 l^4$, $c_{22} = I_{22}l^3$, and $c_{21} = I_{21}l^3$. The numerical values of c_1, c_2, c_{11}, c_{22}, c_{12}, and c_{21} have been given in Ref. 1. On the basis of Eq. (26) stability has been analyzed numerically and stability boundaries determined on a digital computer by evaluating the following determinant and its minors:

$$\Delta = \begin{vmatrix} B_1 & B_0 & 0 & 0 & 0 & 0 \\ B_3 & B_2 & B_1 & B_0 & 0 & 0 \\ B_5 & B_4 & B_3 & B_2 & B_1 & B_0 \\ 0 & B_6 & B_5 & B_4 & B_3 & B_2 \\ 0 & 0 & 0 & B_6 & B_5 & B_4 \\ 0 & 0 & 0 & 0 & 0 & B_6 \end{vmatrix}$$

Stability requires that they be all positive, and stability boundaries are determined by sign reversals. For the two special values of $T = 0$ and ∞, direct calculation of the stability boundary was further made by letting $\Lambda = iL$ and writing from Eq. (26)

$$B_0L^6 - B_2L^4 + B_4L^2 - B_6 = 0$$

$$B_1L^4 - B_3L^2 + B_5 = 0$$

from which L was then eliminated and K finally solved. The results agree with those given by the digital computer.

The results based on the two-mode approximation (Fig. 1) show a stability region which is a narrow strip lying between $K = 0$ and $K = -3.50$ to -4.98. For booms with lengths of several hundred feet such as have been put into use since Ref. 1 was prepared, the value of K can be much lower than -4.98. Accordingly, *thermal bending flutter can take place when the boom is pointed either away from or towards the sun.* Physically, this is not surprising because the second-cantilever mode involves reversal in curvature. When interacting with the first mode, it can cause phase reversal in the resultant force relationship. The result in Fig. 1 further reveals the existence of a singularity point for stability at $K = -29.4$ when $T = 0$. No physical explanation can be given, but $T = 0$ corresponds to the case of a zero thermal curvature in which the physical problem does not present itself at any rate.

Clearly, we have further come to another conclusion that *thermal flutter of the boom can be strongly influenced by the presence of higher modes.* It will be interesting, and important, to investigate the effects caused by the simultaneous presence of additional modes higher than the first and second that have been considered here.

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